Comment on "Motion of an impurity particle in an ultracold quasi-one-dimensional gas of hard-core bosons"

S. Giraud and R. Combescot

Laboratoire de Physique Statistique, Ecole Normale Supérieure, UPMC Paris 06, Université Paris Diderot, CNRS, 24 rue Lhomond, F-75005 Paris, France

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Very recently Girardeau and Minguzzi [Phys. Rev. A **79**, 033610 (2009)] have studied an impurity in a one-dimensional gas of hard-core bosons. In particular they dealt with the general case where the mass of the impurity is different from the mass of the bosons and the impurity-boson interaction is not necessarily infinitely repulsive. We show that one of their initial steps is unjustified, contradicting known exact results. Their results in the general case apply only actually when the mass of the impurity is infinite.

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Quite recently Girardeau and Minguzzi [1] (GM) have studied the complex problem of an impurity in a one-dimensional gas of hard-core bosons, with contact interaction. After considering the case of impenetrable particles with the impurity mass m_i equal to the boson mass m, they aim to generalize to the case of a different mass impurity with any coupling strength λ. After a boson to fermion mapping, one has to deal with fermions. For the $m_i = m$ case this problem has been completely solved analytically for any λ by McGuire [2]. For any λ we have solved analytically [3] the very simple $m_i = \infty$ case and treated the general mass case with an excellent approximation.

After a transformation to shift the impurity position at the origin, GM end up with the Hamiltonian:

$$\hat{H}_F = -\frac{1}{2m}\int dx\ \hat{\psi}_F^{\dagger}(x)\frac{\partial^2}{\partial x^2}\hat{\psi}_F(x) + \lambda\,\hat{\rho}_F(0) + \frac{\hat{p}_F^2}{2m_i} \quad (1)$$

for a system with zero total momentum.

GM argue that the last term $\hat{p}_F^2/2m_i$ is negligible in the thermodynamic limit, which implies that the detailed physics of the system is completely independent of the impurity mass. However a very heavy impurity with infinitely strong repulsion will act as a fixed infinitely repulsive boundary. By contrast a very light impurity will very strongly push away the fermions in order to be widely delocalized. Clearly the energy required to put the impurity in the Fermi sea is quite different in these two cases. This conclusion is supported by known exact results as we show now.

McGuire [2] has solved the $m_i = m$ case for any λ. He has calculated the impurity binding energy E_b (a quantity easy to grasp physically), as well as its effective mass. His result is

$$\frac{E_b}{E_F} = -\frac{1}{\pi}[2y - \pi y^2 + 2(1+y^2)\arctan y], \quad (2)$$

where $y = m\lambda/2k_F$, $E_F = k_F^2/2m$ is the Fermi energy, and k_F is the Fermi wave vector of these fermions.

On the other hand we find for the infinite impurity mass the different result:

$$\frac{E_b}{E_F} = -\frac{1}{\pi}[2y - 2\pi y^2 + (1+4y^2)\arctan 2y]. \quad (3)$$

Hence the impurity mass is relevant for the physics of the system.

GM have calculated explicitly the impurity-fermion distribution function $\rho(x-y)$. Their calculations correspond only to the $m_i = \infty$ case. We have also calculated [3] this quantity in this case and found an analytical expression as a simple integral, plotted in our Fig. 13 for various λ (not all the same as GM). When λ is the same, our results agree with GM. In particular when $\lambda = \infty$ we obtain $\rho(x) = 1 - \sin(2k_F x)/(2k_F x)$, in agreement with their result. McGuire [2] has also calculated $\rho(x)$ for equal masses $m_i = m$ and shown plots for several values of λ. A very striking feature (see Ref. [13] in [3]) is that $\rho(x) \leqslant 1$ in the repulsive case, while $\rho(x) \geqslant 1$ in the attractive one, in striking contradiction with GM. In particular McGuire obtains $\rho(x) = 1 - [\sin(k_F x)/(k_F x)]^2$ for $\lambda = \infty$, different from the above result for $m_i = \infty$.

Finally GM justify their omission of the $\hat{p}_F^2/2m_i$ term by stating that the average of this term is $O(1)$. This is obviously correct. Hence it is definitely negligible in the thermodynamic limit compared to the total ground state energy, which is merely the energy of a free Fermi sea. However the physics of $O(L)$ is completely trivial and uninteresting because it is the physics of a free Fermi sea. All the interesting quantities in this problem are of order $O(1)$, just like the binding energy E_b, Eq. (2) or Eq. (3), and they have to be kept. The final suggestion that the contribution to $\rho(x)$ could be $O(L^{-1})$ is disproved by the explicit examples given in the preceding paragraph.

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[1] M. D. Girardeau and A. Minguzzi, Phys. Rev. A **79**, 033610 (2009).

[2] J. B. McGuire, J. Math. Phys. (NY) **6**, 432 (1965); **7**, 123 (1966).

[3] S. Giraud and R. Combescot, Phys. Rev. A **79**, 043615 (2009).

